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Rates of neutrino conversion and decay in hot QED plasma

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Abstract

Using a real-time formalism of thermal field theory, we derive the reaction-rate formula for neutrino-conversion ($\nu \to \nu'$) process and $\nu\bar{\nu}'$ annihilation process, which take place in an ultrarelativistic QED plasma consisting of electrons, positrons, and photons. Also derived is the formula for the inverse processes to the above ones. On the basis of the hard-thermal-loop resummation scheme, we include the contribution from the coherent processes. For the case of isotropic neutrino distribution, numerical computation is carried out for the parameter region of type-II super-nova explosion. Defferential reaction rate exhibits characteristic peak structure, which comes from the coherent processes. The contribution from the above processes to the decay or damping rate of a parent massive neutrino ν is also studied.

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1 Introduction

For the past two decades, properties of neutrinos in background media have attracted much interest (see e.g., [1]). Interactions of neutrinos with a thermal background cause a change in the properties of neutrinos. The dispersion relation is the quantity that describes this change. Nötzold and Raffelt [2] were the first who comprehensively analyzed the dispersion relation of a neutrino in a thermal background, where, among others, the damping rate of an electron neutrino is computed, the rate which is related to the mean-free path and to the refractive index. Computation is performed by neglecting Pauli blocking effects and using the bare dispersion relation for participating electrons. Radiative decay of a massive neutrino has been analyzed in [3].

It is by now well known [4, 5] that, in hot QED, the thermal propagators of a soft photon[4] and a soft electron (positron) are drastically changed from those of respective bare counterparts. The salient feature is the appearance of the imaginary part for space-like-momentum region, which comes from Landau damping mechanism. The dispersion relations for soft photon and electron are also largely changed. An effective or improved perturbation theory, called hard-thermal-loop (HTL) resummation scheme [4, 5], in which the above-mentioned effects are taken into account, is established just after the work [2].

Recently, in relation to possible neutrino oscillation, neutrino–conversion processes have attracted much interest. In this paper, we deal with a neutrino-conversion ($\nu \to \nu'$) process and a $\nu\bar{\nu}'$ annihilation process, which take place in a hot QED (or electron–positron–photon) plasma. We also deal with inverse processes to them. On the basis of the effective perturbation theory of hot QED, we derive the reaction-rate formula for these processes. For the purpose of illustration, numerical computation is carried out for the differential reaction rate for the case of isotropic neutrino distribution. The contribution from the coherent processes exhibits a characteristic peak structure in energy distribution of a "decay neutrino" — ν' for the neutrino-conversion process and $\bar{\nu}'$ for the $\nu\bar{\nu}'$ annihilation process. We then study the contribution to the damping rate of ν. Concrete computation is carried out for the case where no neutrino exists in background. We are interested in the temperature and baryon-number density regions of type-II supernova explosion (cf. [4, 6]); $m_e << T,\ \mu << m_{ion}$. As in [3],

[4] A soft particle is the particle that carries soft momentum Q^μ ($|Q^\mu| = O(e\sqrt{T^2+\mu^2})$). Here $-e$ is the electron charge and T (μ) denotes the temperature (chemical potential).

we neglect the effect of the ions.

2 Reaction-rate formula and damping rate of a massive neutrino

We deal with the system that consists of a hot QED plasma and neutrinos. We assume that the hot QED plasma is in thermal and chemical equilibrium, while the neutrinos are not. A neutrino-conversion process of our concern is

$$\nu(K) + \text{hot QED plasma} \to \nu'(K') + \text{anything}. \tag{1}$$

We assume that ν is massive, left-handed neutrino (with mass m), ν' is massless and left-handed. The four-momenta K and K' are $K = (E, \mathbf{k})$ with $E = \sqrt{k^2 + m^2}$ and $K' = (k', \mathbf{k}')$, respectively. The hot QED plasma is assumed to be at rest. The total reaction rate for the process (1) contributes to the damping or "decay" rate Γ_d of a parent neutrino ν. Γ_d also receives a contribution from the relative process to (1),

$$\nu(K) + \bar{\nu}'(-K') + \text{hot QED plasma} \to \text{anything}. \tag{2}$$

Here $\bar{\nu}'$ is an antiparticle of ν' and $-K' = (k', -\mathbf{k}')$.

The energy and mass regions of our interest are

$$E << M_W \quad \text{and} \quad m < 2m_e, \tag{3}$$

respectively. Here M_W (m_e) is the mass of W boson (electron). Then, we may use the effective Lagrangian, which, after Fierz transformation, reads

$$\mathcal{L}_{\text{eff}} = -2\sqrt{2}G \left[\left(\underline{\bar{\nu}}' \gamma^\mu L \underline{\nu} \right) \left(\underline{\bar{e}} \gamma_\mu L \underline{e} \right) + \text{h.c.} \right].$$

Here $L \equiv (1 - \gamma_5)/2$, G is Fermi's constant, and underlined fields stand for the fields *in the "weak-interaction basis"*.

For the QED sector, we employ the real-time formalism of thermal QED [4, 7]. In the region of our interest, $T, \mu, E, k' << M_W$, the $O(G^2)$ contribution to Γ_d, $\Gamma_d^{(1)}$, reads

$$\begin{aligned}
\Gamma_d^{(1)} &= |U_{e\nu}^* U_{e\nu'}|^2 \, \tilde{\Gamma}_d^{(1)}, &(4)\\
\tilde{\Gamma}_d^{(1)} &= \frac{i}{2E} \int \frac{d^4Q}{(2\pi)^3} \left[\theta(E - q_0) - n_{\nu'}(K - Q) \right] \\
&\quad \times \delta((K-Q)^2) \, \mathrm{Tr} \left[(\not{K} + m) \, \gamma_\alpha \, L \, (\not{K} - \not{Q}) \, \gamma_\beta \, L \right] \\
&\quad \times \Pi_{21}^{(W)\alpha\beta}(Q). &(5)
\end{aligned}$$

The diagram for $\tilde{\Gamma}_d^{(1)}$ is shown in Fig. 1. In Eq. (4), U is the lepton mixing matrix and, in Eq. (5), $Q \equiv K - K' = (q_0, \mathbf{q})$. $n_{\nu'}(K - Q)$ $(= n_{\nu'}(K'))$ is related to the distribution functions of ν' and of $\bar{\nu}'$ through

$$\begin{aligned} n_{\nu'}(K') &= \theta(k_0')N_{\nu'}(k_0', \hat{\mathbf{k}}') + \theta(-k_0')N_{\bar{\nu}'}(-k_0', -\hat{\mathbf{k}}') \\ &\qquad\qquad\qquad\qquad (\hat{\mathbf{k}}' \equiv \mathbf{k}'/k'), \end{aligned}$$

where $N_{\nu'}(k_0', \hat{\mathbf{k}}')$ $[N_{\bar{\nu}'}(-k_0', -\hat{\mathbf{k}}')]$ is the distribution function of ν' $[\bar{\nu}']$ with momentum $\mathbf{k}'$ $[-\mathbf{k}']$. $\Pi_{21}^{(W)\alpha\beta}(Q)$ in Eq. (5) is the $(2\,1)$-component of the one-loop thermal "self-energy-part" matrix of W:

$$\begin{aligned} \Pi_{ij}^{(W)\alpha\beta}(Q) &= 8iG^2(-)^{i+j} \int \frac{d^4P}{(2\pi)^4} \,\mathrm{Tr}\,[S_{ji}(P-Q) \\ &\qquad \times \gamma^\alpha L S_{ij}(P)\,\gamma^\beta L] \\ &\qquad\qquad (i, j = 1, 2), \end{aligned} \tag{6}$$

where no summation is taken over i and j, and S_{ij} is the $(i\,j)$-component of the bare thermal propagator matrix of an electron, whose form is displayed in Appendix A. The region $q_0 \leq E$ $[q_0 > E]$ of $\tilde{\Gamma}_d^{(1)}$ in Eq. (5) represents the reaction rate for the process (1) [(2)].

We decompose $\Pi_{ij}^{(W)\alpha\beta}(Q)$, Eq. (6), into transverse (T), longitudinal (L), and vector–axial-vector interference (VA) parts and write

$$\begin{aligned} \Pi_{ij}^{(W)\alpha\beta}(Q) &= 4\frac{G^2}{e^2}\Big[\mathcal{P}_T^{\alpha\beta}(\hat{\mathbf{q}})\,\Pi_{ij}^{(T)}(Q) + \mathcal{P}_L^{\alpha\beta}(Q)\,\Pi_{ij}^{(L)}(Q) \\ &\qquad -i\epsilon^{\alpha\beta\rho 0}Q_\rho \Pi_{ij}^{(VA)}(Q)\Big], \end{aligned} \tag{7}$$

$$\begin{aligned} \Pi_{ij}^{(VA)}(Q) &= 2ie^2\frac{Q^2}{q^2}\int \frac{d^4P}{(2\pi)^4}(q_0 - 2p_0) \\ &\qquad \times \tilde{S}_{ji}(P-Q)\tilde{S}_{ij}(P), \end{aligned} \tag{8}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is a fully anti-symmetric pseudo tensor with $\epsilon^{0123} = 1$, $\tilde{S}_{ij}$ $(i, j = 1, 2)$ are as in Eqs. (A.2) and (A.3), and $\mathcal{P}_{T(L)}^{\alpha\beta}$ is the standard projection operator onto transverse (longitudinal) mode,

$$\begin{aligned} \mathcal{P}_T^{\alpha\beta}(\hat{\mathbf{q}}) &= -\sum_{i,j=1}^{3} g^{\alpha i}g^{\beta j}(\delta^{ij} - \hat{q}^i\hat{q}^j), \\ \mathcal{P}_L^{\alpha\beta}(Q) &= g^{\alpha\beta} - \frac{Q^\alpha Q^\beta}{Q^2} - \mathcal{P}_T^{\alpha\beta}(\hat{\mathbf{q}}). \end{aligned} \tag{9}$$

$\Pi_{ij}^{(T/L)}(Q)$ in Eq. (7) is the (ij)-element of the T/L-component of the real-time thermal self-energy part of a photon in hot QED.

$\Pi_{21}^{(S)}(Q)$ $(S = T, L, VA)$ are related to [4, 7] the so-called Feynman self-energy part

$$\begin{aligned} \Pi_F^{(S)}(Q) &\equiv \Pi_{11}^{(S)}(Q) + \theta(q_0)\Pi_{12}^{(S)}(Q) + \theta(-q_0)\Pi_{21}^{(S)}(Q) \\ &\qquad\qquad\qquad (S = T, L, VA), \end{aligned} \tag{10}$$

through

$$\begin{aligned} i\Pi_{21(12)}^{(S)}(Q) &= 2[\theta(\pm q_0) + n_B(|q_0|)]\,\mathrm{Im}\Pi_F^{(S)}(Q) \\ &\qquad\qquad\qquad (S = T, L, VA). \end{aligned} \tag{11}$$

Here $n_B(x) = 1/(e^{x/T} - 1)$ and 'Im' means to take the imaginary part with Feynman prescription. In Appendix B, $\Pi_F^{(S)}$ $(S = T, L, VA)$ is computed within the approximation $m_e = 0$, which is a good approximation for a plasma with high temperature and/or density, $m_e << T, \mu$. In Sec. III, we discuss to what extent the approximation $m_e = 0$ is good one. It is worth mentioning that $\Pi_F^{(VA)}(Q)$ vanishes for vanishing chemical potential, $\mu = 0$.

Straightforward manipulation of Eq. (5) using Eqs. (6) - (11) yields

$$\tilde{\Gamma}_d^{(1)} = \int_{-\infty}^{+\infty} dq_0\, \frac{d\tilde{\Gamma}_d^{(1)}}{dq_0}, \tag{12}$$

$$\begin{aligned} \frac{d\tilde{\Gamma}_d^{(1)}}{dq_0} &= \frac{G^2}{\pi^2}\frac{1}{e^2}\frac{1}{Ek}\,[\theta(q_0) + n_B(|q_0|)]\,\epsilon(E - q_0) \\ &\quad \times \int_{\mathcal{R}} dq\, q\, [\theta(E - q_0) - n_{\nu'}(K - Q)]\, G^{(1)}(Q), \end{aligned} \tag{13}$$

where

$$\begin{aligned} G^{(1)}(Q) &= \epsilon(E - q_0)\,\mathrm{Im}\,\Big[H_T(Q)\Pi_F^{(T)}(Q) \\ &\quad + H_L(Q)\Pi_F^{(L)}(Q) - H_{VA}(Q)\,\Pi_F^{(VA)}(Q)\Big], \end{aligned} \tag{14}$$

$$\begin{aligned} H_T(Q) &= Q^2 - 2k^2 - m^2 + \frac{2}{q^2}\left(Eq_0 - \frac{m^2 + Q^2}{2}\right)^2, \\ H_L(Q) &= 2k^2 + \frac{m^2}{2} - \frac{m^4}{2Q^2} - \frac{2}{q^2}\left(Eq_0 - \frac{m^2 + Q^2}{2}\right)^2, \\ H_{VA}(Q) &= q_0(Q^2 + m^2) - 2EQ^2, \end{aligned} \tag{15}$$

The integration region $\mathcal{R}$ in Eq. (13) is defined as (see Fig. 2)

$$
\begin{aligned}
\mathcal{R} &= \mathcal{R}_1 \cup \mathcal{R}_2, \\
\mathcal{R}_1 &: \ |E-k-q_0| \le q \le E+k-q_0, \\
\mathcal{R}_2 &: \ |E+k-q_0| \le q \le -E+k+q_0.
\end{aligned}
$$

In the region $\mathcal{R}_1$ [$\mathcal{R}_2$], $k'_0 = E - q_0 \ge 0$ [$k'_0 < 0$], and then $\mathcal{R}_1$ [$\mathcal{R}_2$] is the kinematically allowed region of the reaction (1) [(2)]. At first sight, at $q_0 = E - k$, Eq. (13) seems to diverge at $q = 0$. Inspection of the formulae in Appendix B tells us, however, that this is not the case.

We now turn to the inverse processes to (1) and to (2):

$$
\nu'(K') + \text{hot QED plasma} \to \nu(K) + \text{anything}, \tag{16}
$$

$$
\text{hot QED plasma} \to \nu(K) + \bar{\nu}'(-K') + \text{anything}. \tag{17}
$$

The process (16) is a production process of ν due to the reaction of ν' with constituents of the QED plasma (e, e^+, and γ) and the process (17) is a $\nu\bar{\nu}'$ production process. In a similar manner as above, we obtain the reaction-rate formula for these processes,

$$
\Gamma_p^{(1)} = |U_{e\nu}^* U_{e\nu'}|^2 \, \tilde{\Gamma}_p^{(1)}, \tag{18}
$$

$$
\begin{aligned}
\tilde{\Gamma}_p^{(1)} &= \frac{i}{2E} \int \frac{d^4Q}{(2\pi)^3} \left[\theta(q_0 - E) - n_{\nu'}(K-Q)\right] \\
&\quad \times \delta((K-Q)^2) \, \mathrm{Tr}\left[(\not{K} + m)\, \gamma_\alpha \, L \, (\not{K} - \not{Q})\, \gamma_\beta \, L\right] \\
&\quad \times \Pi_{12}^{(W)\alpha\beta}(Q) \\
&= \int_{-\infty}^{+\infty} dq_0 \, \frac{d\tilde{\Gamma}_p^{(1)}}{dq_0},
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\frac{d\tilde{\Gamma}_p^{(1)}}{dq_0} &= \frac{G^2}{\pi^2} \frac{1}{e^2} \frac{1}{Ek} \left[\theta(-q_0) + n_B(|q_0|)\right] \epsilon(q_0 - E) \\
&\quad \times \int_{\mathcal{R}} dq \, q \, \left[\theta(q_0 - E) - n_{\nu'}(K-Q)\right] G^{(1)}(Q),
\end{aligned} \tag{20}
$$

where $G^{(1)}(Q)$ is as in Eq. (14). The diagram for $\tilde{\Gamma}_p^{(1)}$ is the same as Fig. 1, provided that the two types of vertices are interchanged, $1 \leftrightarrow 2$.

The net decay rate Γ_d^{net} is

$$
\Gamma_d^{\mathrm{net}}(E, \hat{\mathbf{k}}) = n_\nu(K) \Gamma_d(E, \hat{\mathbf{k}}) - [1 - n_\nu(K)] \Gamma_p(E, \hat{\mathbf{k}}), \tag{21}
$$

where $n_\nu(K) = N_\nu(E, \hat{\mathbf{k}})$ is a distribution function of ν. When ν, ν', and $\bar{\nu}'$ are in thermal and chemical equilibrium, n_ν and $n_{\nu'}$ take similar form to Eq. (A.4). Using

Eq. (11), one can show, in this case, that the detailed balance holds, $\Gamma_d^{\rm net}(E,\hat{\mathbf{k}}) = \Gamma_d^{\rm net}(E) = 0$.

According to the HTL-resummation scheme [5, 4], the integration region in Eqs. (12), (13), (19), and (20) should be divided into hard-Q region ($|Q^\mu| \sim \sqrt{T^2+\mu^2}$) and the soft-$Q$ region ($|Q^\mu| = O(\sqrt{T^2+\mu^2})$).

Hard-Q region: For ${\rm Im}\Pi_F^{(S)}(Q)$ ($S = T, L, VA$), expressions given in Appendix B are used.

Soft-Q region: Observing the formulae in Appendix B, we see that, for $e << 1$,

$$
\begin{aligned}
H_{T/L}(Q)\,{\rm Im}\Pi_F^{(T/L)}(Q) &\simeq H_{T/L}(Q)\,{\rm Im}F_{T/L}(Q) \\
&>> H_{VA}(Q)\,\Pi_{21}^{(VA)}(Q), \qquad (22)
\end{aligned}
$$

where ${\rm Im}F_{T/L}(Q)$ is as in Eq. (B.1) with Eqs. (B.14) and (B.15) in Appendix B. In the soft-Q region, there is an additional contribution: An inverse HTL-resummed photon propagator (cf. Eq. (A.7)) $\left({}^*\Delta_F^{(T/L)}(Q)\right)^{-1}$ $(= Q^2 - \Pi_F^{(T/L)}(Q) \simeq Q^2 - F_{T/L}(Q))$ is of the same order of magnitude as $\Pi_F^{(T/L)}(Q)$ $(\simeq F_{T/L}(Q))$. Thus, the diagram for $\tilde{\Gamma}_d$ as shown in Fig. 3 yields an equally important contribution.

The characteristic scale parameter of the hard region is [5, 4] $\sqrt{T^2+\mu^2}$, and that of the soft region is $e\sqrt{T^2+\mu^2}$. As a matter of fact, observing that $e \simeq 0.30$, the hard region and the soft region are not *sharply separated*. Taking this fact into account, we compute the contribution from Fig. 3, $\Gamma^{(2)}$, without using the HTL-approximation ($e << 1$). The contribution is given by Eq. (5) with the replacement [4, 7],

$$
\begin{aligned}
\Pi_{21}^{(W)\alpha\beta}(Q) &\to -\frac{e^2}{8G^2}\sum_{i,j=1}^{2}\Pi_{2i}^{(W)\alpha\mu}(Q) \\
&\quad \times \left({}^*\Delta_{ij}(Q)\right)_{\mu\nu}\Pi_{j1}^{(W)\nu\beta}(Q), \qquad (23)
\end{aligned}
$$

where Π's are as in Eq. (6) and ${}^*\Delta$ is as in Appendix A.2. Straightforward computation yields

$$
\begin{aligned}
\frac{d\tilde{\Gamma}_d^{(2)}}{dq_0} &= \frac{G^2}{\pi^2}\frac{1}{e^2}\frac{1}{Ek}\left[\theta(q_0) + n_B(|q_0|)\right]\epsilon(E-q_0) \\
&\quad \times \int_{\mathcal{R}} dq\, q\, \left[\theta(E-q_0) - n_{\nu'}(K-Q)\right] G^{(2)}(Q), \qquad (24)
\end{aligned}
$$

where

$$G^{(2)}(Q) = \frac{1}{2}\epsilon(E-q_0)\,\mathrm{Im}\left[\sum_{P=T,L} H_P(Q)\frac{\left(\Pi_F^{(P)}(Q)\right)^2}{Q^2-\Pi_F^{(P)}(Q)} + H_T(Q)q^2\,\frac{\left(\Pi_F^{(VA)}(Q)\right)^2}{Q^2-\Pi_F^{(T)}(Q)} - 2H_{VA}(Q)\,\frac{\Pi_F^{(T)}(Q)\Pi_F^{(VA)}(Q)}{Q^2-\Pi_F^{(T)}(Q)}\right]. \tag{25}$$

The replacement (23) and Fig. 3 tell us [8] that Eq. (24) describes the differential rate for a set of processes, in which real and/or virtual photon(s) participate. It is to be noted that the (real) photons in the QED plasma are in thermal equilibrium. Then, the photon(s) does not come out of the plasma, so that, when the decay neutrino goes out from the plasma, it does not accompany photon(s). [In this relation, see [3]].

In a similar manner, we obtain, for the contribution from the processes (16) and (17),

$$\frac{d\tilde{\Gamma}_p^{(2)}}{dq_0} = \frac{G^2}{\pi^2}\frac{1}{e^2}\frac{1}{Ek}\left[\theta(-q_0)+n_B(|q_0|)\right]\epsilon(q_0-E) \times \int_{\mathcal{R}} dq\,q\,\left[\theta(q_0-E)-n_{\nu'}(K-Q)\right]G^{(2)}(Q). \tag{26}$$

The diagram for this is the same as Fig. 3, provided that the type-1 vertex and the type-2 vertex in Fig. 3 are interchanged.

In the next section, we shall use the formulae displayed above for the whole Q^2-region.

3 Numerical computation

We are interested in the type-II supernova environment, which is a QED plasma whose core temperature is $T \sim 30-60$ MeV and electron chemical potential is $\mu \sim 350$ MeV [4].

3.1 Differential reaction rates

In general, the ν'-distribution function $n_{\nu'}(K') = n_{\nu'}(k_0', \hat{\mathbf{k}}')$ [$K' = K - Q$] is anisotropic, and one should compute Eqs. (13), (20), (24), and (26) substituting given

$n_{\nu'}(K')$. In this section, we restrict ourselves to the case of isotropic distribution, $n_{\nu'}(k'_0, \hat{\mathbf{k}}') = n_{\nu'}(k'_0)$. In this case, Eqs. (13), (20), (24), and (26) may be written in the form,

$$
\begin{aligned}
\frac{d\tilde{\Gamma}_d^{(i)}}{dq_0} &= \frac{G^2}{\pi^2}\frac{1}{e^2}\frac{1}{Ek}\epsilon(E-q_0)\left[\theta(E-q_0) - n_{\nu'}(E-q_0)\right] \\
&\quad \times \mathcal{G}^{(i)}(q_0, E, T) \qquad\qquad (i = 1, 2), \\
\frac{d\tilde{\Gamma}_p^{(i)}}{dq_0} &= \frac{G^2}{\pi^2}\frac{1}{e^2}\frac{e^{-q_0/T}}{Ek}\epsilon(q_0 - E) \\
&\quad \times \left[\theta(q_0 - E) - n_{\nu'}(E-q_0)\right]\mathcal{G}^{(i)}(q_0, E, T) \\
&\qquad\qquad\qquad (i = 1, 2).
\end{aligned}
$$

Here

$$
\mathcal{G}^{(i)}(q_0, E, T) \equiv \left[\theta(q_0) + n_B(|q_0|)\right]\int_{\mathcal{R}} dq\, q\, G^{(i)}(Q) \qquad (i = 1, 2)
$$

with $G^{(1)}(Q)$ and $G^{(2)}(Q)$ as in Eqs. (14) and (25), respectively. We compute $\mathcal{G}^{(i)}(q_0, E, T)$ $(i = 1, 2)$ for various values for the parameters T, E, and m. For the chemical potential, unless otherwise stated, we take $\mu = 350$ MeV [4]. We find that the numerical results are insensitive to the neutrino mass m at least in the range $0 \leq m \lesssim 100$ eV/c^2 [9]. This is natural since m ($\lesssim 100$ eV/c^2) is much smaller than all other parameters, T, μ, and E. We take $m = 0.01$ eV/c^2 throughout.

In Figs. 4 - 8, we display the results of numerical computation for different values for E and T. The solid lines represent the total contributions $\mathcal{G} \equiv \mathcal{G}^{(1)} + \mathcal{G}^{(2)}$, while the dot-dashed lines represent $\mathcal{G}^{(1)}$. The figures "(a)" display $\mathcal{G}$ and $\mathcal{G}^{(1)}$ in the region $q_0 \leq E$ [the region of the processes (1) and (16)] and the figures "(b)" display $\mathcal{G}$ and $\mathcal{G}^{(1)}$ in the region $q_0 > E$ [the region of the processes (2) and (17)]. Some observations are in order.

- Figures 4 - 7 show the results for different values of E with $T = 50$ MeV. We see, as is expected, that for smaller incident-neutrino energy E, $\mathcal{G}^{(2)}/\mathcal{G}$ is larger. In the region of figures (b) [the region of the processes (2) and (17)], both $\mathcal{G}^{(1)}$ and $\mathcal{G}^{(2)}$ ($= \mathcal{G} - \mathcal{G}^{(1)}$) are positive. In the region of figures (a) [the region of the processes (1) and (16)], except for the small region $q_0 \sim 0$ (or $k' \sim E$) in the case of relatively small E/T, $\mathcal{G}^{(2)}$ is negative. Referring to the reaction-rate formula [8], one can see what kind of physical processes are involved in $d\tilde{\Gamma}_d^{(2)}/dq_0$

(Eq. (24)) and $d\tilde{\Gamma}_p^{(2)}/dq_0$ (Eq. (26)). As a matter of fact, each of them involves a set of infinite number of coherent processes. A few examples of them that are involved in $d\tilde{\Gamma}_d^{(2)}/dq_0$ are

$$\nu + e \to e + \gamma + \nu' \,, \qquad \nu + e \to e + e + e^+ + \nu'.$$

Figures 4 - 8 tell us that, for most regions displayed in figures (a), an infinite number of "interference contributions" is summed up to be negative, so that $d\tilde{\Gamma}_d^{(2)}/dq_0$ and $d\tilde{\Gamma}_p^{(2)}/dq_0$ are negative.

- Both in figures (a) and (b), $\mathcal{G}^{(2)}(q_0, E, T)$ exhibits peak structure. For figures (a) $[q_0 < E]$, the peak is at $q_0 \simeq 0$ or $k' = E - q_0 \simeq E$ and is more prominent for smaller incident energy E. The structure of figures (b) $[q_0 \geq E]$ may be understood as follows. In the hard-thermal-loop approximation [cf. Eq. (22)], ${}^*\Delta_F^{(T/L)}(Q) \simeq 1/[Q^2 - F_{T/L}(Q)]$ (see Eqs. (A.7) and (B.1)). Then, as is well known or as can be shown from Eq. (11) with Eqs. (B.3) and (B.4), ${}^*\Delta_{T/L}^{(12)/(21)}(Q)$ in Eq. (A.6) turns out to be of the form

$$\begin{aligned} {}^*\Delta_{T/L}^{(12)/(21)}(Q) &= 2i[\theta(\mp q_0) + n_B(|q_0|)] Z_{T/L}(q) \\ & \quad \times \delta(q_0 - \omega_{T/L}(q)) \qquad (q_0 > q). \end{aligned} \tag{27}$$

The dispersion curves, $q_0 = \omega_T(q)$ and $q_0 = \omega_L(q)$, are shown in Fig. 2. Use of the actual ${}^*\Delta_F^{T/L}(Q) = 1/[Q^2 - \Pi_F^{T/L}(Q)]$ results in the change of $\delta(q_0 - \omega_{T/L}(q))$ in Eq. (27) to the functions with finite width that are (sharply) peaked at $q_0 \simeq \omega_{T/L}(q)$. Inspection of Fig. 2 with these observation in mind allows us to understand the structure of figures (b).

For the purpose of seeing the effect of the chemical potential μ, we display in Fig. 9 the result for $(E, T, \mu) = (10, 50, 0)$ MeV. [For the QED plasma in the early universe, $\mu \simeq 0$.] We see that $\mathcal{G}^{(2)} << \mathcal{G}^{(1)}$, so that the peak structure is less prominent when compared to the case of $\mu = 350$ MeV, Fig. 5. $\mathcal{G}$ in the region $q_0 > E$ is much larger than $\mathcal{G}$ in the region $q_0 < E$.

Above computation is carried out neglecting the electron mass m_e. Inclusion of the electron mass m_e causes a change in $\Pi_F^{(S)}(Q)$ $(S = T, L, VA)$ in the region $|Q^2| \leq O(m_e^2)$. For the purpose of getting a measure to what extent the approximation $m_e = 0$ is good one, we perform all numerical computations by simply cutting off the region $|Q^2| < m_e^2$. This cutoff turns out to reduce $\mathcal{G}^{(i)}$ $(i = 1, 2)$. Dashed lines in Figs. 4 - 9 show the result of computation. In most regions of Figs. 4 - 9, no substantial reduction arises. Especially, for the region $q_0 \geq E$, no sizable reduction arises and we do not display the results in figures (b). For the region $q_0 < E$, prominent reduction occurs only at $q_0 \simeq 0$, at which $\mathcal{G}^{(2)}$ peaks. Larger reduction occurs for smaller E.

3.2 Decay rate

For computing the contributions to the decay or damping rate $\tilde{\Gamma}_d$ ($= \tilde{\Gamma}_d^{(1)} + \tilde{\Gamma}_d^{(2)}$) (cf. Eq. (12)) and to the production rate $\tilde{\Gamma}_p$ ($= \tilde{\Gamma}_p^{(1)} + \tilde{\Gamma}_p^{(2)}$) (see Eq. (19)), knowledge for the distribution function $n_{\nu'}(K - Q)$ is necessary. Furthermore, for computing the net decay rate $\Gamma_d^{\rm net}(E, \hat{\mathbf{k}})$, Eq. (21), knowledge for the distribution function $n_\nu(K)$ is necessary.

Here we compute the damping rate $\tilde{\Gamma}_d$ of an incident ν on a hot QED plasma with no background neutrinos, $n_\nu = n_{\nu'} = 0$. Then, the process (2) is absent. Displayed in Figs. 10 and 11 are the total contribution $\tilde{\Gamma}_d$ ($= \tilde{\Gamma}_d^{(1)} + \tilde{\Gamma}_d^{(2)}$) and the partial contribution $\tilde{\Gamma}_d^{(1)}$ for $\mu = 350$ MeV and, in respective order, $T = 50$ MeV and 20 MeV. Figures 12 and 13 show the result for $\mu = 0$ and, in respective order, $T = 50$ MeV and 20 MeV.

Cutting off the contribution from the region $|Q^2| < m_e^2$ does not result in sizable reduction.

Characteristic features:

- For the range of E, T, and μ displayed in Figs. 10 - 13, the contribution from the soft-Q region, $\tilde{\Gamma}_d^{(2)}$, is not very large.

- Figures 12 and 13 tell us that, for $\mu = 0$, $\tilde{\Gamma}_d$ is almost linear in E. As a matter of fact, $\tilde{\Gamma}_d$'s in Figs. 12 and 13 are well parametrized as

$$\tilde{\Gamma}_d(E, T) = cG^2 E^{1+\alpha} T^{4-\alpha}$$

  with $(c, \alpha) = (0.60, 0.04)$.

4 Discussions

For relativistic particles dealt with here, the mean-free path l is related to the decay rate Γ through $l = 1/\Gamma$ [2], which, in turn, is related to the imaginary part of the refractive index $\mathrm{Im}[n] = (2lE)^{-1} = \Gamma/2E$. In the range of Figs. 10 and 11, order of magnitude of $\tilde{\Gamma}_d$ is $10^{-15} \sim 10^{-12}$ MeV. Then, we see from Eq. (4) that $l \simeq [0.2 \sim 200]/|U_{e\nu}^* U_{e\nu'}|^2$ m, which is much less than the core size of the type-II supernova. This means that, when applying to the actual supernova, $\bar{\nu}\nu'$ as well as $\nu\bar{\nu}'$

production processes are also important and, through these processes, (anti)neutrinos are produced. Thus, one confronts with the necessity of more elaborate analysis, in which the distribution functions of (anti)neutrinos and their evolution through the Boltzmann equation are taken into account. Furthermore, we have assumed in this paper that the hot QED plasma is in thermal and chemical equilibrium and is of infinite size. More realistic treatment for this is also necessary.

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Appendix A Thermal propagators

1.1 Bare thermal propagator of a massless electron

Elements of the bare thermal electron propagator matrix reads [4, 7]

$$\begin{aligned} S_{ij}(P) &= \tilde{S}_{ij}(P)\not{P} \qquad (i,j=1,2), & \text{(A.1)} \\ \tilde{S}_{11}(P) &= -\left(\tilde{S}_{22}(P)\right)^* \\ &= \frac{1}{P^2+i0^+} + 2\pi i n_e(p_0)\delta(P^2), & \text{(A.2)} \\ \tilde{S}_{12(21)}(P) &= -2\pi i[\theta(\mp p_0) - n_e(p_0)]\,\delta(P^2), & \text{(A.3)} \end{aligned}$$

where

$$n_e(p_0) = \theta(p_0)\frac{1}{e^{(p_0-\mu)/T}+1} + \theta(-p_0)\frac{1}{e^{(|p_0|+\mu)/T}+1}. \tag{A.4}$$

Here T is the temperature of the QED plasma and μ is the chemical potential being conjugate to the electron number.

1.2 Effective thermal propagator of a soft photon

Elements of the effective soft-photon propagator matrix (in Landau gauge) reads [4, 5]

$$\begin{aligned} {}^*\Delta_{ij}^{\alpha\beta}(Q) &= -\mathcal{P}_T^{\alpha\beta}(\hat{\mathbf{q}})\,{}^*\Delta_T^{(ij)}(Q) - \mathcal{P}_L^{\alpha\beta}(Q)\,{}^*\Delta_L^{(ij)}(Q) \\ & \qquad (i,j=1,2), \end{aligned} \tag{A.5}$$

where $\mathcal{P}_T^{\alpha\beta}(\hat{\mathbf{q}})$ and $\mathcal{P}_L^{\alpha\beta}(Q)$ are as in Eq. (9) and

$$\begin{aligned}
{}^*\Delta_{T/L}^{(11)}(Q) &= -\left({}^*\Delta_{T/L}^{(22)}(Q)\right)^* \\
&= {}^*\Delta_F^{(T/L)}(Q) + 2in_B(|q_0|)\,\mathrm{Im}{}^*\Delta_F^{(T/L)}(Q), \\
{}^*\Delta_{T/L}^{(12)/(21)}(Q) &= 2i[\theta(\mp q_0) + n_B(|q_0|)]\,\mathrm{Im}{}^*\Delta_F^{(T/L)}(Q), \qquad \text{(A.6)} \\
{}^*\Delta_F^{(T/L)}(Q) &= \frac{1}{Q^2 - \Pi_F^{(T/L)}(Q)}. \qquad \text{(A.7)}
\end{aligned}$$

$\Pi_F^{(T/L)}$ is computed in Appendix B.

Appendix B Thermal self-energy part $\Pi_F^{(S)}$ $(S = T, L, VA)$

Here we compute the lowest-order contribution to $\Pi_F^S(Q)$ $(S = T, L, VA))$, Eq. (10). We are interested in the high-T and large-μ region $T, \mu >> m_e$, and then we ignore m_e. The effect of m_e $(\neq 0)$ is discussed in Sec. III.

Computation of $\Pi_F^{(S)}(Q)$ $(S = T, L, VA)$

We decompose $\Pi_F^{(T)}(Q)$ and $\Pi_F^{(L)}(Q)$ into three parts,

$$\Pi_F^{(T/L)}(Q) = F_{T/L}^{(0)}(Q) + F_{T/L}(Q) + G_{T/L}(Q). \qquad \text{(B.1)}$$

$F_{T/L}^{(0)}$ stands for the vacuum contribution and $F_{T/L}$ stand for the contributions that dominate in the soft-Q region, the latter contributions which are called hard thermal loop [4, 5]. Incidentally, $\Pi_F^{(VA)}(Q)$ has no hard thermal loop.

Straightforward computation of Eq. (10) (cf. Eqs. (6) - (9)) using Eqs. (A.1) - (A.3) yields

$$\begin{aligned}
F_T^{(0)}(Q) &= F_L^{(0)}(Q) = -\frac{\alpha}{3\pi}Q^2\left[\frac{5}{3} - \ln\left(\frac{-Q^2}{\mu_r^2}\right)\right], \qquad \text{(B.2)} \\
F_T(Q) &= \frac{3}{2}\,m_\gamma^2\frac{q_0}{q}\left[\frac{q_0}{q} - \frac{Q^2}{2q^2}\,\ln\frac{q_0+q}{q_0-q}\right], \qquad \text{(B.3)} \\
F_L(Q) &= -3\,m_\gamma^2\frac{Q^2}{q^2}\left[1 - \frac{q_0}{2q}\,\ln\frac{q_0+q}{q_0-q}\right], \qquad \text{(B.4)} \\
G_T(Q) &= -\frac{\alpha}{\pi}\,Q^2\left[\left(1 + \frac{Q^2}{2q^2}\right)I_1 + \frac{2}{q^2}\,(I_2 - q_0 I_3)\right], \qquad \text{(B.5)} \\
G_L(Q) &= \frac{\alpha}{\pi}\,\frac{Q^2}{q^2}\left[Q^2 I_1 + 4\,(I_2 - q_0 I_3)\right], \qquad \text{(B.6)}
\end{aligned}$$

and

$$\Pi_F^{(VA)} = -\frac{e^2}{4\pi^2}\frac{Q^2}{q^3}\left[\mu q_0 \ln\frac{q_0+q}{q_0-q} - q q_0 \tilde{I}_3 + 2q\tilde{I}_1\right]. \tag{B.7}$$

In obtaining the vacuum contribution (B.2), we have used the $\overline{\rm MS}$ scheme and μ_r is the renormalization scale, for which we choose $\mu_r = \sqrt{T^2+\mu^2}$. [We have adopted a convention that Dirac gamma matrices are 4×4 matrices in D-dimensional spacetime.] Incidentally, the vacuum part of $\Pi_F^{(VA)}$ vanishes. In the above equations, $\alpha = e^2/4\pi$, $m_\gamma^2 = e^2(T^2+3\mu^2/\pi^2)/9$ is the thermal mass of an electron, and

$$\begin{aligned}
I_1 &\equiv -\frac{1}{2q}\int_0^\infty dp[n_+(p)+n_-(p)]\,\ln\left(\frac{L_{++}\,L_{--}}{L_{+-}\,L_{-+}}\right), && \text{(B.8)}\\
I_2 &\equiv \frac{\pi^2}{6}\left(T^2+\frac{3\mu^2}{\pi^2}\right)\\
&\quad -\frac{1}{2q}\int_0^\infty dp\,p^2\,[n_+(p)+n_-(p)]\,\ln\left(\frac{L_{++}\,L_{--}}{L_{+-}\,L_{-+}}\right), && \text{(B.9)}\\
I_3 &\equiv \frac{1}{2q}\int_0^\infty dp\,p\,[n_+(p)+n_-(p)]\,\ln\left(\frac{L_{++}\,L_{+-}}{L_{-+}\,L_{--}}\right), && \text{(B.10)}\\
\tilde{I}_1 &\equiv -\frac{1}{2q}\int_0^\infty dp\,p\,[n_+(p)-n_-(p)]\,\ln\left(\frac{L_{++}\,L_{--}}{L_{+-}\,L_{-+}}\right), && \text{(B.11)}\\
\tilde{I}_3 &\equiv \frac{1}{2q}\int_0^\infty dp\,[n_+(p)-n_-(p)]\,\ln\left(\frac{L_{++}\,L_{+-}}{L_{-+}\,L_{--}}\right) && \text{(B.12)}
\end{aligned}$$

with

$$\begin{aligned}
n_\pm(p) &= 1/(e^{(p\mp\mu)/T}+1),\\
L_{\rho\sigma} &\equiv q_0 + \rho q + 2\sigma p \qquad (\rho,\sigma=\pm).
\end{aligned}$$

It is straightforward to obtain

$$
\begin{aligned}
\mathrm{Im}\Pi_F^{(T/L)}(Q) &\equiv \frac{1}{2i}\left[\Pi_F^{(T/L)}(q_0(1+i\epsilon),q)-\mathrm{c.c.}\right], && \text{(B.13)}\\
\mathrm{Im}F_T(Q) &= \theta(-Q^2)\,\frac{3\pi}{4}\,m_\gamma^2\,Q^2\,\frac{|q_0|}{q^3}, && \text{(B.14)}\\
\mathrm{Im}\left(\frac{F_L(Q)}{Q^2}\right) &= -\theta(-Q^2)\,\frac{3\pi}{2}\,m_\gamma^2\,\frac{|q_0|}{q^3}, && \text{(B.15)}\\
\mathrm{Im}\,I_1 &= -\frac{\pi T}{2q}\left(F_{--}+F_{+-}-F_{-+}-F_{++}\right), && \text{(B.16)}\\
\mathrm{Im}\,I_2 &= -\frac{\pi}{2q}\int_{q_l}^{q_u} dp\,p^2[n_+(p)+n_-(p)], && \text{(B.17)}\\
\mathrm{Im}\,I_3 &= -\frac{\pi}{2q}\epsilon(q_0)\int_{q_l}^{q_u} dp\,p[n_+(p)+n_-(p)] \\
&\quad -\frac{\pi}{q}\epsilon(q_0)\theta(-Q^2) \\
&\quad \times\int_0^{q_l} dp\,p\,[n_+(p)+n_-(p)], && \text{(B.18)}\\
\mathrm{Im}\tilde{I}_1 &= -\frac{\pi}{2q}\int_{q_l}^{q_u} dp\,p\,\left[n_+(p)-n_-(p)\right], && \text{(B.19)}\\
\mathrm{Im}\tilde{I}_3 &= \frac{\pi}{q}\epsilon(q_0)\theta(-Q^2)\mu \\
&\quad -\frac{\pi}{2q}\epsilon(q_0)T\left[F_{++}-F_{-+}\right. \\
&\quad \left.-\epsilon(Q^2)\left(F_{+-}-F_{--}\right)\right], && \text{(B.20)}
\end{aligned}
$$

where

$$
\begin{aligned}
q_u &\equiv \frac{|q_0|+q}{2}, \qquad q_l \equiv \frac{||q_0|-q|}{2}, \\
F_{\rho\sigma} &= \ln\left(e^{\rho\mu/T}+e^{-||q_0|+\sigma q|/(2T)}\right) \qquad (\rho,\sigma=+,-). && \text{(B.21)}
\end{aligned}
$$

Note that $\Pi_F^{(VA)}(Q)$ vanishes for $\mu=0$.

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FIG. 1. Diagram for $\tilde{\Gamma}_d^{(1)}$. "1" ("2") at the vertex on the left-side (right-side) denotes the type of vertex in real-time nonequilibrium quantum field theory.

FIG. 2. Integration region $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$. The dashed line with T (L) shows the dispersion relation for the transverse (longitudinal) mode in the hard-thermal-loop resummed photon propagator.

FIG. 3. Diagram for $\tilde{\Gamma}_d^{(2)}$. The hard-thermal-loop resummed effective photon propagator is indicated by a blob. "1", "2", "i", and "j" on the vertexes denote the type of vertex.

FIG. 4. Plots of $\mathcal{G}$ and $\mathcal{G}^{(1)}$ vs q_0 at $T = 50$ MeV, $\mu = 350$ MeV, and $E = 2$ MeV. Figure (a) corresponds to the processes (1) and (16), and Fig. (b) corresponds to the processes (2) and (17).

FIG. 5. Same as in Fig. 4 but for $E = 10$ MeV.

FIG. 6. Same as in Fig. 4 but for $E = 20$ MeV.

Fig. 7. Same as in Fig. 4 but for $E = 50$ MeV.

Fig. 8. Same as in Fig. 4 but for $T = 20$ MeV and $E = 10$ MeV.

Fig. 9. Same as in Fig. 4 but for $\mu = 0$ and $E = 10$ MeV.

Fig. 10. Plots of $\tilde{\Gamma}_d$ and $\tilde{\Gamma}_d^{(1)}$ vs E at $T = 50$ MeV and $\mu = 350$ MeV.

Fig. 11. Same as in Fig. 10 but for $T = 20$ MeV.

Fig. 12. Plots of $\tilde{\Gamma}_d$ and $\tilde{\Gamma}_d^{(1)}$ vs E at $T = 50$ MeV and $\mu = 0$.

Fig. 13. Same as in Fig. 12 but for $T = 20$.